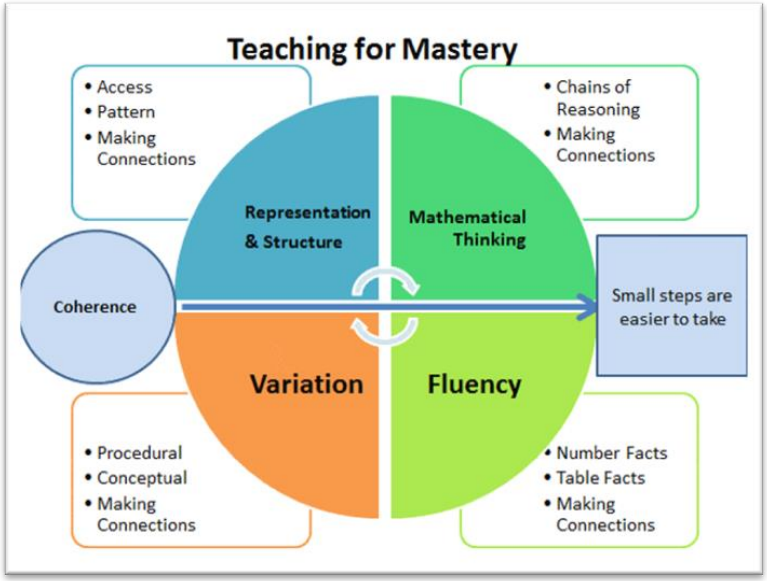
National Curriculum objectives

Year 3	Year 4	Year 5	Year 6
Addition & Subtraction Add and subtract numbers mentally, including: a three-digit number and 1s a three-digit number and 10s a three-digit number and 100s Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction Estimate the answer to a calculation and use inverse operations to check answers Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction **Multiplication & Division** Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables Write and calculate mathematical statements for multiplication and	**Addition & Subtraction** Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate Estimate and use inverse operations to check answers to a calculation Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why **Multiplication & Division** Recall multiplication and division facts for multiplication tables up to 12 × 12 Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers Recognise and use factor pairs and commutativity in mental calculations	**Addition & Subtraction** Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) Add and subtract numbers mentally with increasingly large numbers Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why **Multiplication & Division** Identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long	**Addition, Subtraction, Multiplication & Division** Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context Perform mental calculations, including with mixed operations and large numbers Use their knowledge of the order of operations to carry out calculations

division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	Multiply two-digit and three-digit numbers by a one-digit number using formal written layout Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	multiplication for two-digit numbers Multiply and divide numbers mentally, drawing upon known facts Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 Solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.	involving the 4 operations Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Solve problems involving addition, subtraction, multiplication and division Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

The five concepts of Mastery



1. Coherence

Connecting new ideas to concepts that have already been understood, and ensuring that, once understood and mastered, new ideas are used again in next steps of learning. This is successful when learning is: broken into small, carefully sequenced steps, linked to prior learning and each new step is focussed on in depth so that learning is sustainable.

2. Representation & structure

Representations used in lessons expose the mathematical structure being taught. The more representations taught, the more influence this has on a child's ability to transform, compare and combine numbers when calculating.

3. Variation

Variation within teaching highlights essential features of a concept or idea through varying the non-essential features. Questions such as: 'What is the same?' 'What is different?' encourage the children to identify key structures that must be maintained to understand and apply the concept.

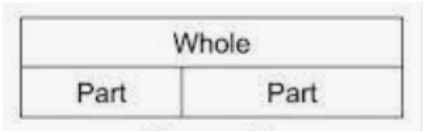
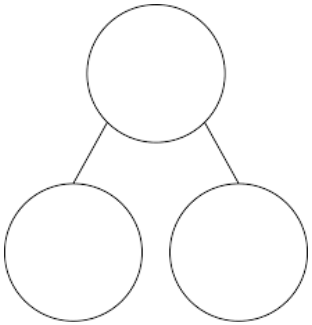
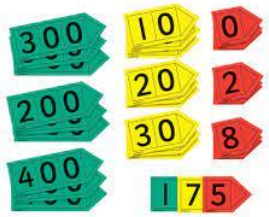
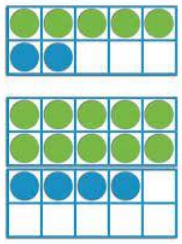
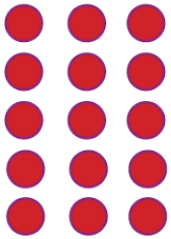
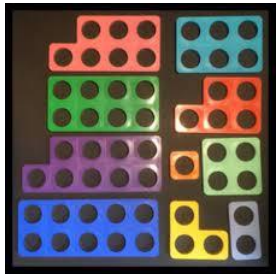
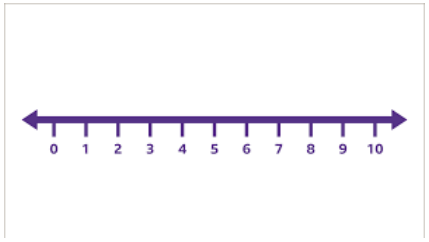
4. Fluency

Fluency demands more of learners than memorisation of a single procedure or collection of facts. It encompasses a mixture of efficiency, accuracy and flexibility. Quick and efficient recall of facts and procedures is important in order for learners to keep track of problems, think strategically and solve problems. It also demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections and to make appropriate choices from a whole toolkit of methods, strategies and approaches.

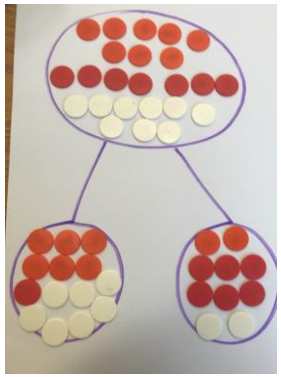
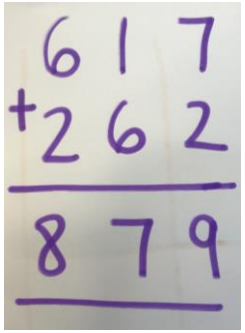
5. Mathematical thinking

If taught ideas are to be understood deeply, they must not merely be passively received but must be worked on by the child: thought about, reasoned with and discussed with others. By explaining, convincing, drawing diagrams and using manipulatives, the children develop a deep and sustainable style of learning, which will lend itself well as children are faced with more challenging, abstract concepts.

Representations and Resources

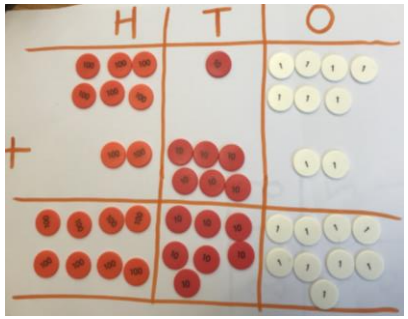
Place value counters/charts 	Cubes 	Bar models 	Part-Whole models 
Arrow cards 	Tens frames 	Arrays 	Numicon 
Dienes	Number lines 	Counting Sticks 	Counters 

	Concrete	Pictorial	Abstract
Year 3 **Addition** add total plus sum more altogether column addition regroup	In examples of addition without regrouping, children are required to show their understanding of presenting their numbers in a column method. Once they have done this, the children will start adding the ones, followed by the tens etc. In these scenarios, the children do not need to regroup any numbers. Once they are confident with this concept, the children are then moved on to the concept of regrouping.	**Addition without regrouping** Here, a part-whole model is being used as a pictorial tool to show that the two smaller bubbles are equal to the larger, central bubble.  Number lines can also be used to help add on smaller amounts. They are also used to help the children to estimate an answer, by rounding and	The knowledge gained through representation can now be used in a column method structure; where numbers are used to represent the place value composition in a compact way.  **Step 1**: Add the numbers in the ones column.

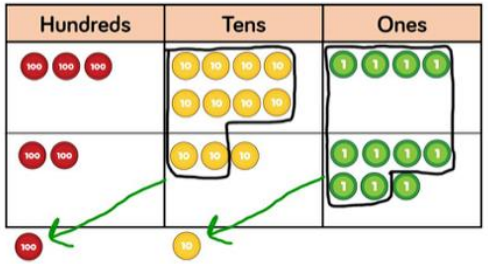
Subtraction

difference
subtract
less
minus
take away
Column subtraction

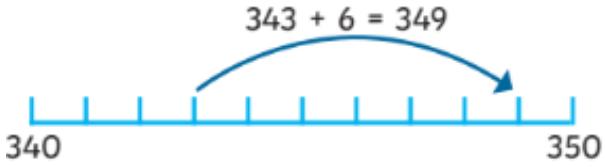


Addition with regrouping

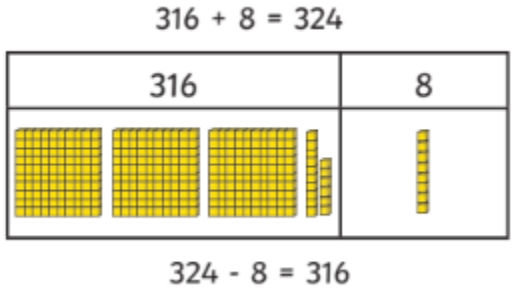
When adding, the children will start by adding the ones. In this instance, 10 ones are being regrouped as 1 ten. This means there is only one left in the ones column and another ten can be added to the tens column. When adding the tens, 10 tens can be regrouped as 1 hundred, therefore adding one more hundred to the hundreds column and leaving 2 tens behind in the tens column.



recognising 'near numbers'.



All children are encouraged to check their answers by using the inverse operation. In the case of addition, subtraction can be used, as seen below:



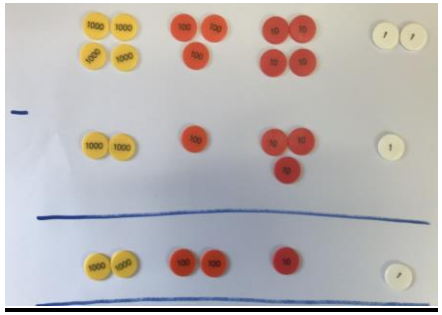
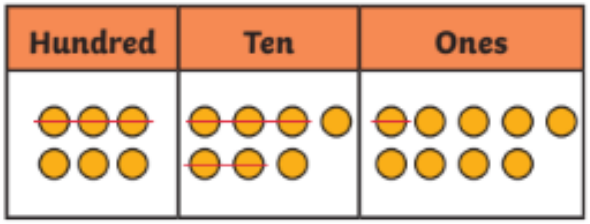
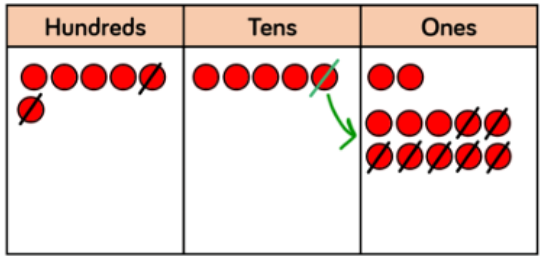
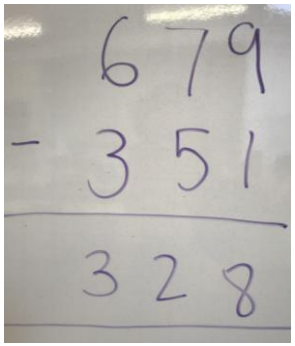
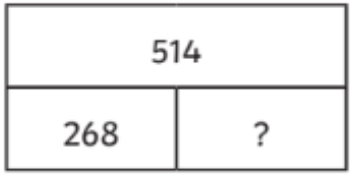
Subtraction without exchanging

Step 2: Add the numbers in the tens column.
Step 3: Add the numbers in the hundreds column.

In each case, recognising that the '1' and '6' in the tens column actually represent 10 and 60, for example.

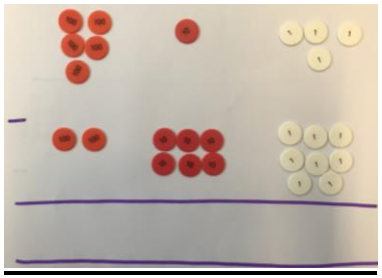
$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ \ 1 \end{array}$$

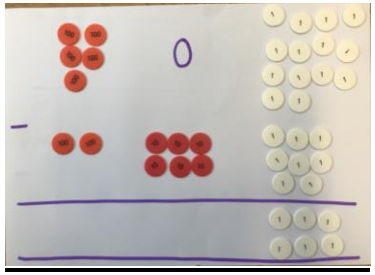
Again, we begin by adding the ones, however, as the total exceeds ten, we regroup ten ones as one ten in the tens column, leaving one in the ones column. This ten is then included in the addition of the tens. In this case, there are 12 tens, therefore, 10 tens can be regrouped as one hundred, leaving 2 tens in the tens column. The regrouped one hundred will be added to the other hundreds, making 6 hundreds in total.

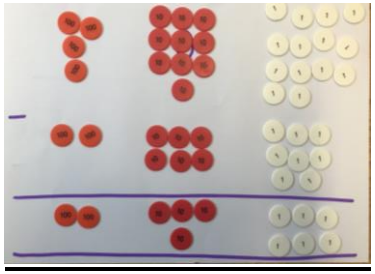
Exchange	 When subtracting, the children need to start by ensuring that the smaller number is being subtracted from the larger number. They will then need to start by subtracting the ones, followed by the tens and the hundreds, as seen with addition. In the case of subtraction with no exchange, the children should be able to use their knowledge of number bonds to ten to help them answer quickly and accurately.	679 – 351 = 328  In this pictorial representation, the children are encouraged to strike-through as they subtract from the total number of ones, tens and hundreds. Although the place value is no longer visible on the individual counters, a place value table is being used to support the child's place value understanding, before progressing on to the abstract method. **Subtraction with exchanging**  Start by looking at the ones column. If the ones in the greater number exceeds the ones digit of the smaller number, then a ten from the tens column	 When laying out a column subtraction, it is vital that the greater number is at the top and the smaller value is below. When presenting their work in a column method, the children are encouraged to ensure clear presentation of each place value column. This reduces calculation errors and gives the children a good presentational foundation to build upon, which they will find beneficial as they progress on to more challenging concepts.  Bar models are used in subtraction problems to demonstrate the 'difference'. It also allows the children to visualise the inverse operation, which will allow them to check their answers.

Multiplication

times tables
multiply by
array
fact families
Regrouping



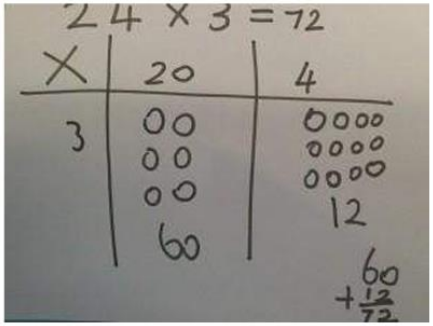


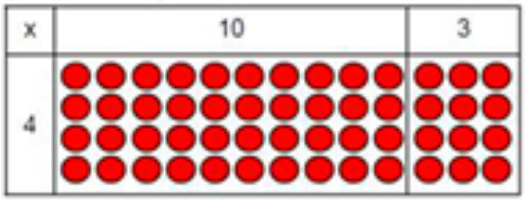


Multiplication

In year 3, the children learn how to multiply 2-digit numbers by 1-digit numbers using the grid method. In order to do this, the children learn how to partition the number into tens and ones so that they can add the products together at the end of the calculation. In the example below, 24 x 3,

needs to be exchanged into ten ones. As seen in the concrete and pictorial representations, the ten is physically moved out of the tens column and becomes ten ones in the ones column. This means that there is now a larger amount to subtract from. As the subtraction problems become more challenging, children may face more than one exchange in a single subtraction. If there are not enough tens to subtract from, a single hundred will be exchanged into ten tens, meaning that a hundred will move out of the hundreds column and be moved into the tens column as ten tens. This is demonstrated step-by-step in the concrete representation.





Division by grouping and sharing

$$\begin{array}{r} {}^{4}\cancel{6}\,{}^{1}\cancel{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$

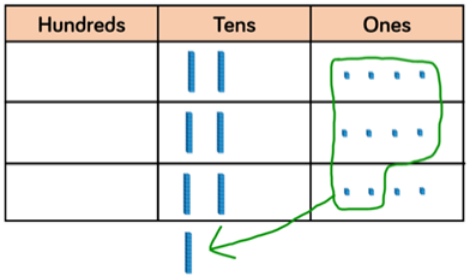
×	30	5
7	210	35

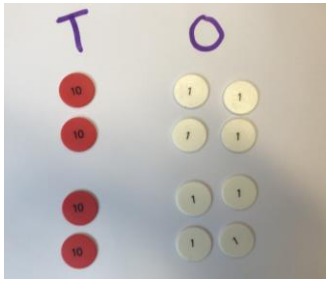
In this instance, 35 has been partitioned into 3 tens and 5 ones allowing each individual component to be multiplied, before the products are added together to find the total.
It is during this time that consolidation of multiplying by ten is crucial to ensure that the children's calculations are accurate. The children are taught this using place value grids, where they are required to move the number 'one space to the left'. Once this has been done, a place holder (0) is required to fill the ones column, showing the movement and multiplication of the original number.

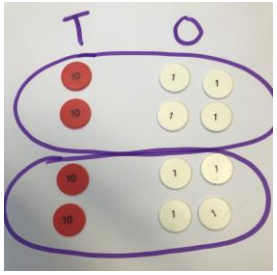
Division

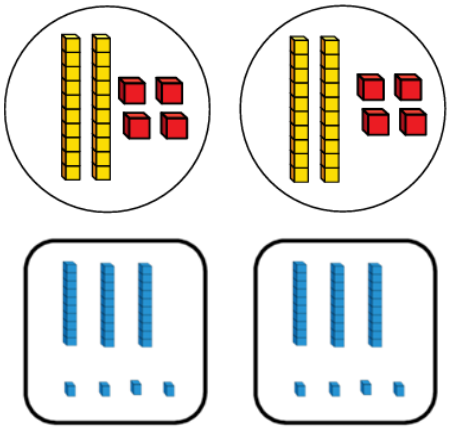
divide by
fact families
share equally
group equally

we have 12 ones that, when added to 6 tens, is equal to 72. The ten ones have been regrouped to form a seventh ten.

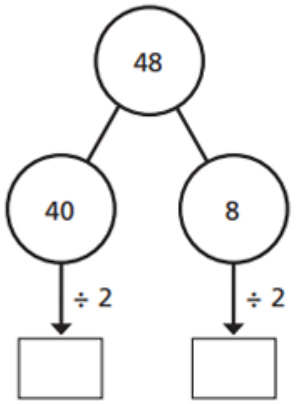








Children will use their knowledge of partitioning to help them to divide equally, recognising that the tens and ones need to be considered as separate values. This concept can be developed further by representing the partitioned number in a part-whole model, as demonstrated in the abstract section.



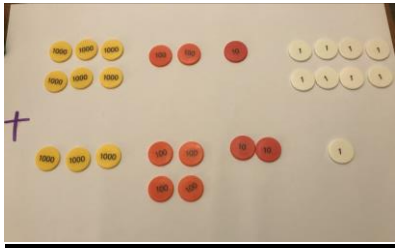
$68 \div 2 = 34$

The children learn that the division sign means to share/group equally. At this stage, the children are not required to represent remainders.

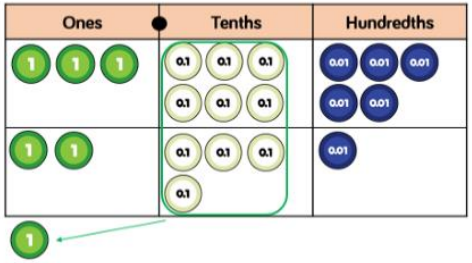
Year 4

Addition

add
total
plus
sum
more
altogether
column addition
regroup

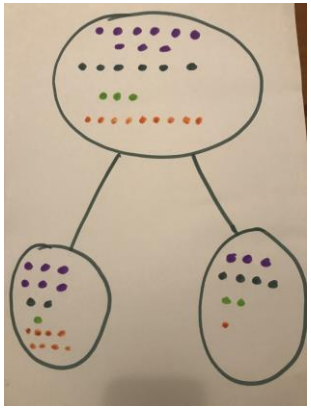


In year 4, the children are required to add 4-digit numbers. When regrouping is not required, the children will be encouraged to solve these problems with increasing speed and accuracy. They may also become increasingly confident at solving these problems mentally.



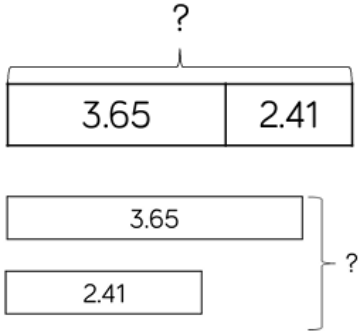
Bar models continue to be used to demonstrate what the total of two or more numbers looks like in representation. It is also used to check for accuracy, using the inverse operation, subtraction. Both scenarios are demonstrated in the pictorial column.

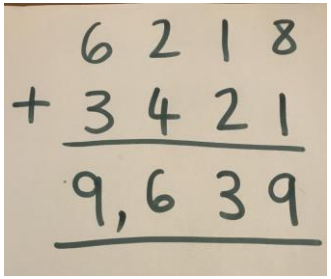
Addition without regrouping



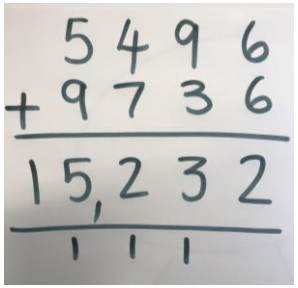
Addition with regrouping

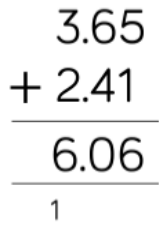
All children will be required to use their previous knowledge of regrouping ones and tens to apply this to regrouping tenths and hundredths when adding amounts of money. They will also regroup hundreds and thousands when adding 3 - and 4-digit numbers.





Commas may be used following the thousands digit to help the children visualise the place value of the total.

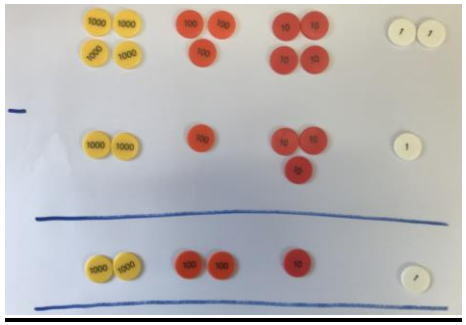




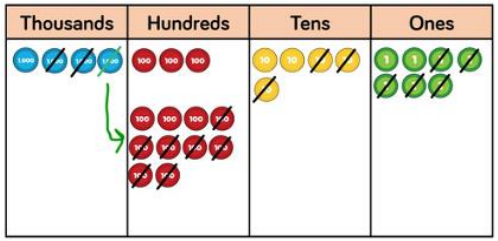
When adding decimals, it is important to understand the purpose of the decimal point and the place value of the tenths and hundredths that follow. Linking this to real-life situations, such as use of

Subtraction

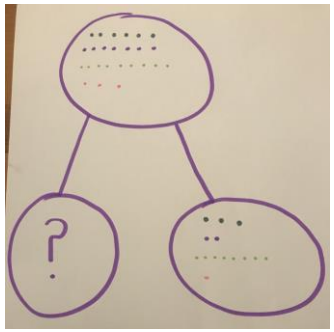
difference
subtract
less
minus
take away
Column subtraction
Exchange



When using place value counters, the children may wish to pick up the counter they are subtracting and cover one from the greater number. Eg. In the ones column, the one being subtracted could be placed on top of a counter in the above number. Therefore 2-1 = 1.



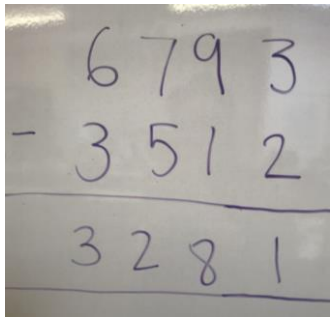
Subtraction without exchanging



In this instance, coloured dots to represent different place values are used in a part-whole model. This representation shows that the difference in values needs to be calculated.

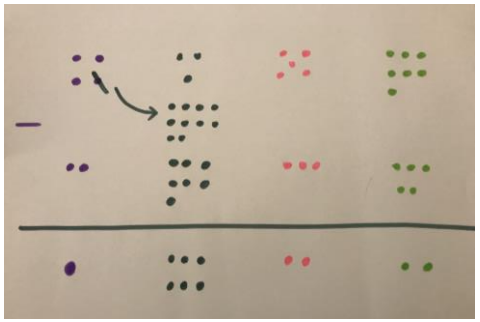
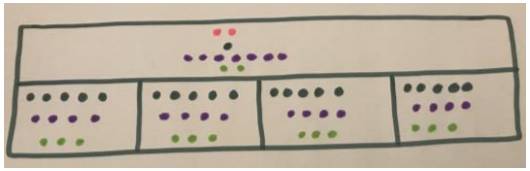
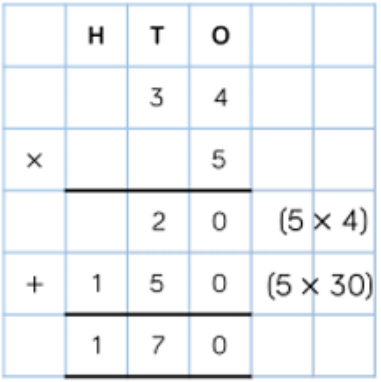
Subtraction with exchanging

money, allows the children to consolidate their understanding of parts of a whole.



As with addition without regrouping, the children are now required to subtract 4-digit numbers. In the case where exchanging is not required, the speed of completion with increased accuracy and greater employment of mental strategies will be encouraged.

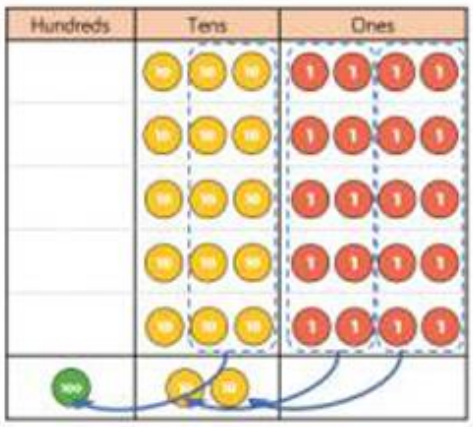
$$\begin{array}{r} {}^{3}\cancel{4}^{1}357 \\ -\ 2735 \\ \hline 1622 \\ \hline \end{array}$$

	When solving subtraction with exchange questions in year 4, the children may find that they are faced with a number of exchanges within the same problem. To support this, the children look back at the place value and how to exchange numbers, particularly now that they may need to exchange a thousand into ten hundreds.	 Using arrows on pictorial representations can remind the children of the physical movement of one thousand into ten hundreds, rather than 'crossing out a digit and taking one away'.	Once confident with the composition of the number, the children can show their calculations as part of a column subtraction. Where they understand that the crossing out of numbers is actually the exchange between place value columns.
Multiplication multiply groups of lots of times factor multiple product	In year 4, the children learn how to multiply a 3-digit number by a 1-digit number, using the expanded column method. By using this method, the composition of the numbers being multiplied is exposed and this, in turn, consolidates the children's understanding of place value within the calculation. The children are required to start by multiplying the ones value, through to the hundreds, by adding each product to a new row. Once the products have been found, their total can be calculated.	**<u>Multiplication</u>**  A bar model is used in this context to show that the product can be represented in four equal parts.	 **Step 1:** The ones are multiplied, 5x4. **Step 2:** The tens value is multiplied, 5 x 30. **Step 3:** The sum of the products is calculated. When writing in the answers, the children are required to show their understanding that the '3' is actually equal to 3 tens.

Division
divide

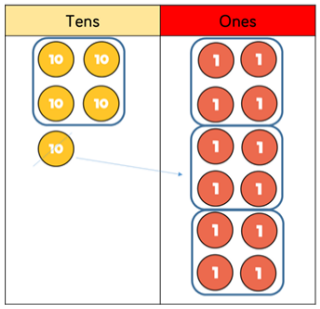
share

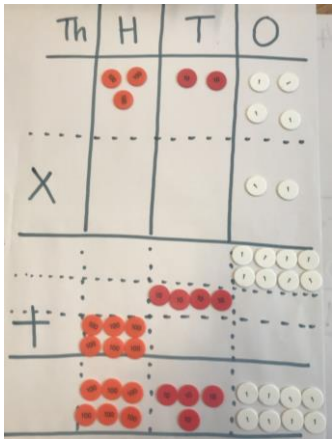
remainder



Once the children are confident in multiplying 2-digit numbers by 1-digit numbers, they will move on to solving problems involving the multiplication of 3-digit numbers by 1-digit numbers.

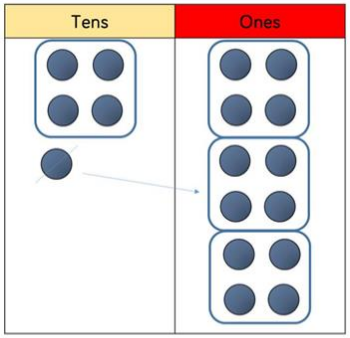
In year 4, the children learn to use the bus-stop method to divide 3-digit numbers by a 1-digit number. In some cases, the children will be required to represent a remainder. At this stage, the children will use 'r' to demonstrate this.





In the example above, the children have demonstrated an understanding of place value by leaving gaps where place holders need to be added in an abstract/written calculation.

Division



By using the bus-stop method, the children are using their times table and related division fact knowledge to complete the question. The more embedded these facts, the quicker and more accurately the problem will be solved.

Th	H	T	O	
	5	4	3	
×			4	
		1	2	(4 × 3)
	1	6	0	(4 × 40)
2	0	0	0	(4 × 500)
2	1	7	2	

		1	3	
	4	5	$^{1}2$	

52

?	?	?	?

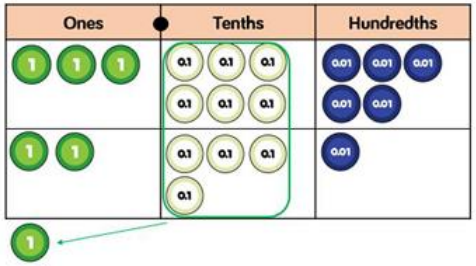
53

13	13	13	13	1

Year 5

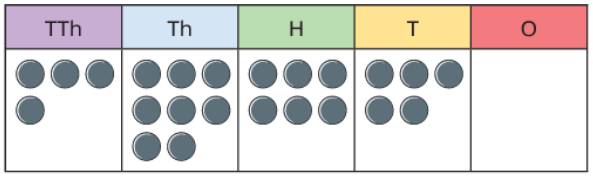
Addition
Add
Total
Make
Plus
Sum
More
Altogether
Column addition
Estimate
Inverse operation

Subtraction
difference
subtract
less
minus
take away
Column subtraction
Exchange



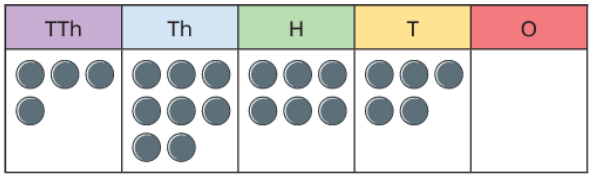
The children will be required to add values of money as part of word problems. Their understanding of being able to regroup tenths and hundredths is essential to be able to reason in these contexts.

Addition

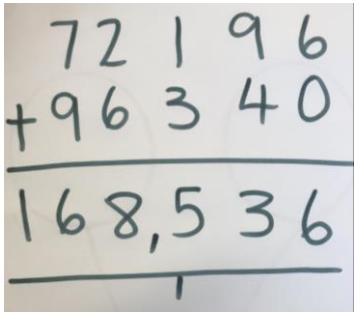


In year 5, the children are required to add 5 digit numbers, where they may need to regroup a series of different numbers to find the answer. At this stage, the children should be confident with the concept of regrouping, as they have covered this in previous year groups. At this stage, the children will also be expected to think more about how they could generate a number that would mean there is one regrouping, two regroupings or more when added to the representation above, for example.

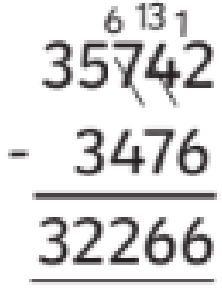
Subtraction



As mentioned above in the addition section, once children are confident with the concept of exchanging within a subtraction calculation, they may be required to generate numbers that would mean there would be one, two or more exchanges within the calculation. This prepares them for questions such as 'missing number' subtractions that are often presented in the column method format.



In upper school, the children continue to use the column method to calculate answers to addition problems involving regrouping. In cases where there are smaller numbers without exchange, the children will be increasingly encouraged to use mental strategies.



Again, the column subtraction method is used. In this instance, the process of exchanging is carried out on two occasions.

Multiplication
multiply
groups of

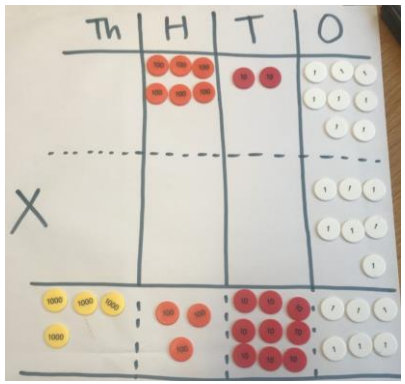
lots of

times

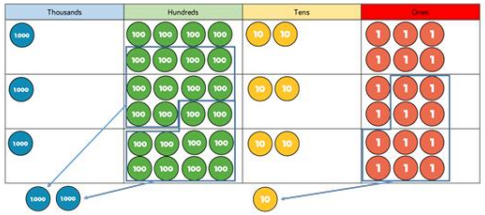
factor

multiple

product

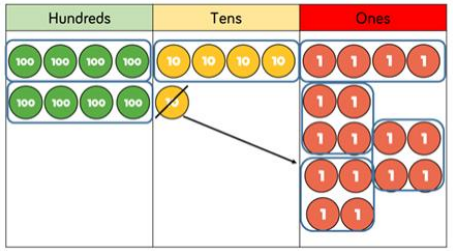


In this example, the children are required to see how a regrouped number is incorporated into the next column, instead of 'layering', as in the expanded method learnt in year 4.

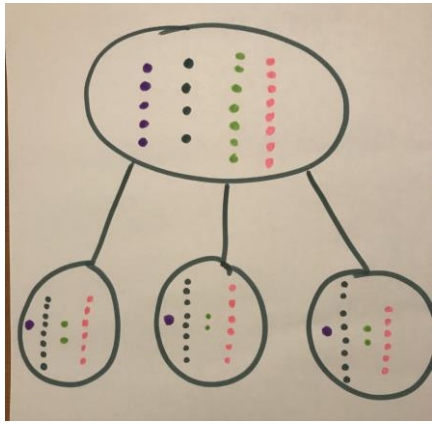


Division

divide
share
remainder



Multiplication



In year 5, the children learn how to use the 'compact' method of multiplication. At first they start by multiplying a 3- or 4-digit number by a 1-digit number, this consolidates their understanding of the compact nature of the method. Once this has been understood, they move on to multiplying by a 2-digit number.

Division

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8

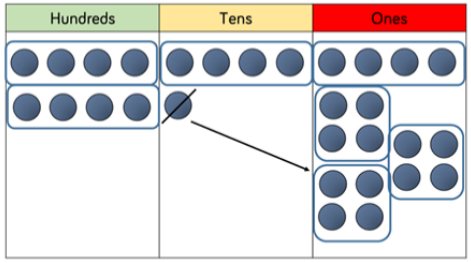
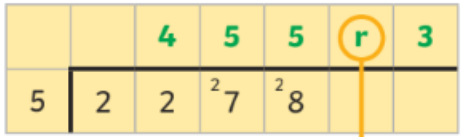
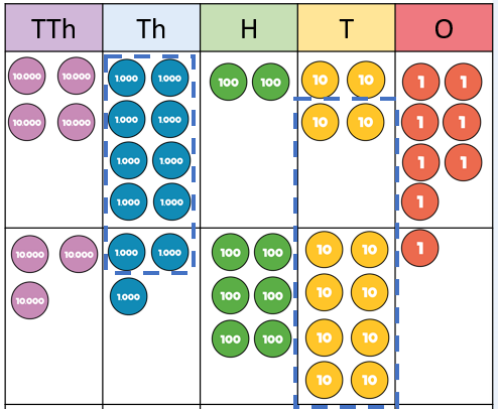
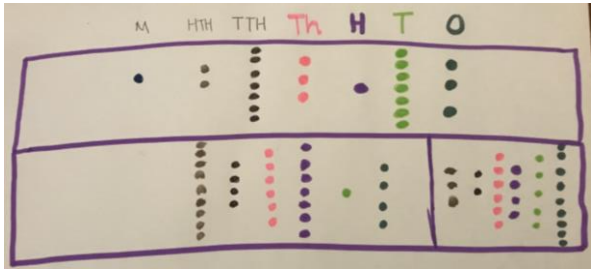
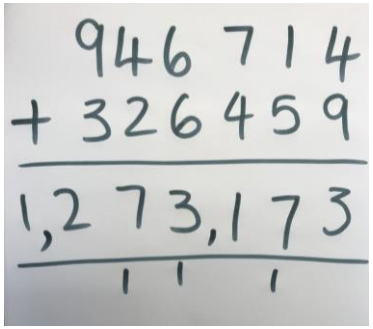
2 1

		2	5	4	3
	×			6	7
	1 1	7 3	8 3	0 2	1
1 1	5 3	2 2	5 1	8	0
1	7	0	3	8	1

1 1

Step 1: Multiply the ones digits (3x7). Followed by 40 x 7, 500 x 7 and 2000 x 7.
Step 2: As the tens value is now being multiplied, by adding a place-holder to the ones column (0), the answer is automatically multiplied by 10. The process is continued by multiplying 6x3, 6x40, 6x500 and 6x2000.
Step 3: The products are added together to find the total.

		2	1	4
	4	8	5	$^{1}6$

	Once again, the children will learn how to exchange numbers that cannot be divided equally as remainders. This is then developed, once again, into using the bus-stop method.	 As in year 4, however, 4- digit numbers will now be used, and the children will be expected to represent their remainders with an 'r' but also develop this into representing their remainders as fractions.	 In this example, the remainder is being indicated using the letter 'r', however, in this instance, the remainder could also be represented as the fraction, ⅗.
<u>Year 6</u> **Addition** Add Total Make Plus Sum More Altogether Column addition Estimate Inverse operation **Subtraction** difference subtract less minus take away Column subtraction Exchange		**<u>Addition</u>**  **<u>Subtraction</u>**	

Multiplication

Multiply

groups of

lots of

times

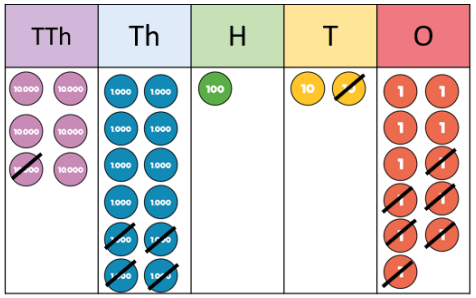
factor

multiple

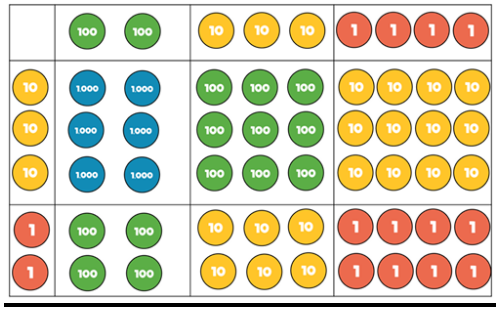
product

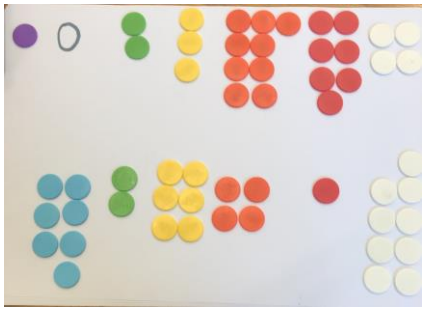
Division

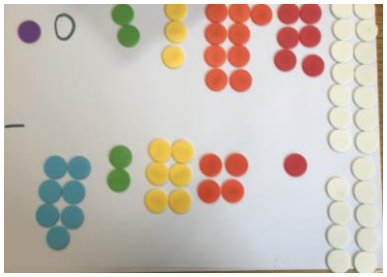
divide



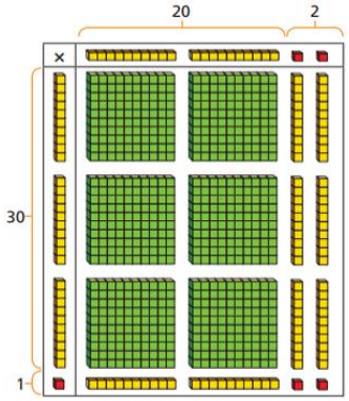
In year 6, children are required to show and explain the reasoning behind the exchange process and be able to demonstrate it through concrete, pictorial and abstract means, as pictured here.



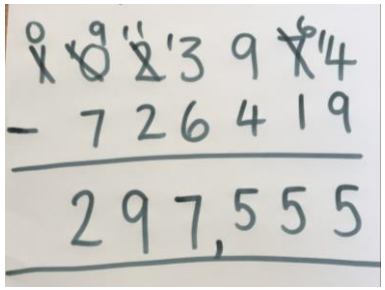




Multiplication



Division



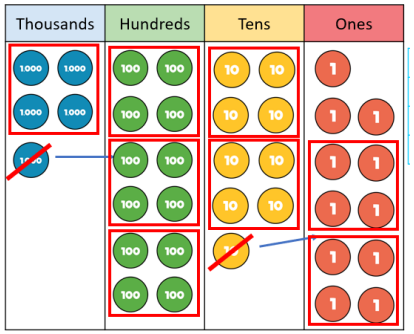
Again, commas are used to indicate thousands and millions, reminding the children of the place value of a number.

Th	H	T	O
	2	3	4
×		3	2
	4	6	8
1 7	1 0	2	0
7	4	8	8

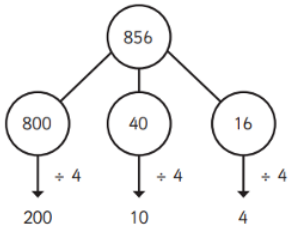
share

remainder

quotient



$856 \div 4 = 214$



	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

		4	4	0	· 5
12	5	$^{5}2$	$^{4}8$	6	$^{\cdot 6}0$

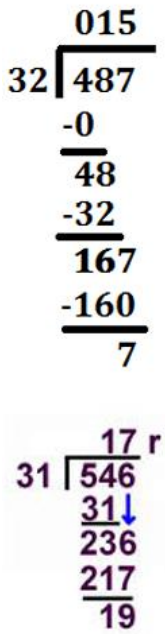
Formal methods of division

Chunking

$7{,}335 \div 15 = 489$

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

Long division

	487 ÷ 32 = 

Order of operations

B	Brackets	$10 \times (4 + 2) = 10 \times 6 = 60$
O	Order	$5 + 2^2 = 5 + 4 = 9$
D	Division	$10 + 6 \div 2 = 10 + 3 = 13$
M	Multiplication	$10 - 4 \times 2 = 10 - 8 = 2$
A	Addition	$10 \times 4 + 7 = 40 + 7 = 47$
S	Subtraction	$10 \div 2 - 3 = 5 - 3 = 2$